Geometrical interpretation of matrices

Walter Trump, 2017-02-03

An ***area matrix*** (A, M) is a combination of a positive matrix A and a set M of continuous graphs.

Main definition

(1) A matrix $A = (a_{ij}) \in \mathbb{R}^{m\times n}$ whose elements are positive real numbers with the total sum S_A combined with a set $M = \{F_0 , F_1 , F_2 , \dots F_m , G_0 , G_1 , G_2 , \dots G_n\}$ of graphs of continuous real functions is called ***area matrix*** if the following conditions are satisfied for $i \in \{0, 1, 2, \dots, m\}$, $j \in \{0, 1, 2, \dots, n\}$ and two positive real numbers x_n and y_m :

(a) $F_i = \{ (x, f_i(x)) \mid x \in \mathbb{R}$ and $0 \le x \le x_n \}$ and $G_j = \{ (g_j(y), y) \mid y \in \mathbb{R}$ and and $0 \le y \le y_m \}$ with $f_i : [0, x_n] \longrightarrow [0, y_m]$ and $g_j : [0, y_m] \longrightarrow [0, x_n]$

(b) $0 = f_0(x) < f_1(x) < f_2(x) < \dots < f_{m-1}(x) < f_m(x) = y_m$ for all $x \in [0, x_n]$ and $0 = g_0(y) < g_1(y) < g_2(y) < \dots < g_{n-1}(y) < g_n(y) = x_n$ for all $y \in [0, y_m]$

(c) $|F_i \cap G_j| = 1$

(d) All functions are differentiable in all points except intersections.

(d) $a_{m+1-i,\, j}$ = area of the locus $\{ (x, y) \mid f_{i-1} (x) \le y < f_i(x)$ and $g_{j-1} (y) \le x < g_j(y) \}$ if $i, j > 0$.

Remarks

The graphs F_0, G_0, F_m, G_n describe the sides of a rectangle with area $e^2 \cdot x_n \cdot y_m = S_A$
with the the size factor $e = \sqrt{\frac{S_A}{x_n \cdot y_m}}$.
This rectangle is partitioned into m·n parts (geometric shapes) by the 2·(n – 1) inner graphs
$F_1 , F_2 , \dots F_{m-1} , G_1 , G_2 , \dots G_{n-1}$.
The area of each part is equal to the associated entry of the matrix.
The graphs $F_0 , F_1 , F_2 , \dots F_m$ do not intersect. The graphs $G_0 , G_1 , G_2 , \dots G_n$ do not intersect.
There are $(m + 1)\cdot(n + 1)$ intersections $P_{ij}(x_{ij} , y_{ij})$ of graphs F_i and G_j .
The connections between neighbor intersections are smooth lines.

This definition was inspired by William Walkington's idea of area magic squares.

(2) An area matrix (A, M) is called ***area square matrix*** if $m = n$ and $x_n = y_m$.

(3) A matrix $A = (a_{ij}) \in \mathbb{N}^{n\times n}$ of order $n > 2$ with n^2 distinct entries is called ***magic square*** with magic sum S if the following conditions are satisfied for all $i, j \in \{0, 1, 2, \dots, n\}$:

(a) $\sum_{k=1}^{n} a_{ik} = \sum_{k=1}^{n} a_{kj} = S$ (sum of the numbers in a row or a column)

(b) $\sum_{k=1}^{n} a_{kk} = \sum_{k=1}^{n} a_{k,(n-k+1)} = S$ (sum of the numbers in two diagonals)

The matrix A is called ***classical magic square*** if the entries are 1, 2, 3, ... n^2.
The matrix A is called ***semi-magic square*** if condition (b) is not satisfied.

(4) An area matrix (A, M) is called ***area magic square*** if the matrix is a magic square.

Further definitions which are also agreed upon with William Walkington

(5) An area matrix (*A*, *M*) is called ***orthogonal*** if all graphs of *M* are constant.
(The rectangle is partitioned into m·n rectangles.)

(6) An area matrix (*A*, *M*) is called ***linear*** if all graphs of *M* are linear.
(The rectangle is partitioned into m·n convex quadrilaterals.)

(7) An area matrix (*A*, *M*) is called ***affine*** if all graphs F_i and G_i are linear in each interval between two consecutive intersections on each graph.
That means: Two neighbor intersections are connected with a straight line segment, the shortest possible connection. The Latin word „affinis“ means „connected with“.
(The rectangle is partitioned into m·n quadrilaterals – not necessarily convex.)

(8) An area matrix (*A*, *M*) is called ***convex*** if all parts are convex.
(Each convex area matrix is affine, but not all affine area matrices are convex.)

Further definitions which are not generally agreed upon

(9) An area matrix (*A*, *M*) is called ***semi-affine*** if all graphs F_i or all graphs G_j are linear in each interval between two consecutive intersections on each graph F_i or each graph G_i.

(10) An area matrix (*A*, *M*) is called ***semi-linear*** if all graphs F_i or all graphs G_j are linear.

(11) An area matrix (*A*, *M*) is called ***semi- orthogonal*** if all graphs F_i or all graphs G_j are constant.
(In a semi-constant linear area matrix the rectangle is partitioned into m·n trapezoids (trapeziums).)